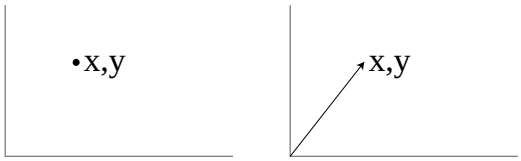
# mas110: recitation 0: vectors

## vectors (points)

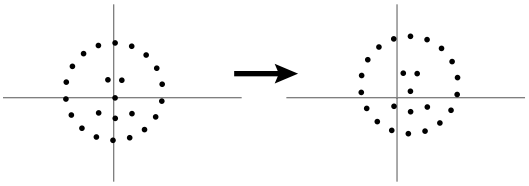
$$V = \begin{bmatrix} x \\ y \end{bmatrix}$$




## addition (translation)

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1+x_2 \\ y_1+y_2 \end{bmatrix} = \begin{bmatrix} x_2+x_1 \\ y_2+y_1 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

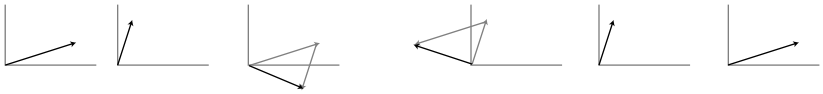
$$U + V = W = W = V + U$$



## subtraction (difference)

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} - \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1-x_2 \\ y_1-y_2 \end{bmatrix} \neq \begin{bmatrix} x_2-x_1 \\ y_2-y_1 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} - \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

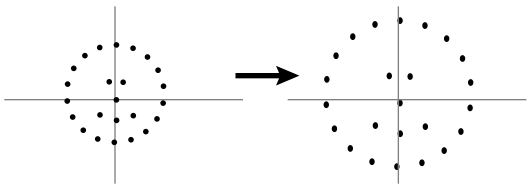
$$U - V = W_0 \neq W_1 = V - U$$



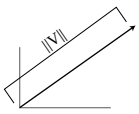
## multiplication (scaling)

$$a\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} ax_1 \\ ay_2 \end{bmatrix}$$

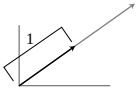
$$aV = W$$



## distance: $\sqrt{x^2+y^2} = \|V\|$



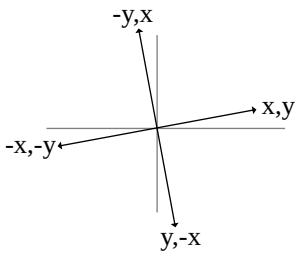

## normalization: $\hat{U} = \frac{U}{\|U\|}$ $\hat{U}$ is a unit vector, $\|\hat{U}\| = 1$



## parallel & perpendicular

$$\begin{bmatrix} x \\ y \end{bmatrix} \parallel \begin{bmatrix} -x \\ -y \end{bmatrix} \perp \begin{bmatrix} y \\ -x \end{bmatrix} \parallel \begin{bmatrix} -y \\ x \end{bmatrix}$$

$$U \parallel -U \perp V \parallel -V$$

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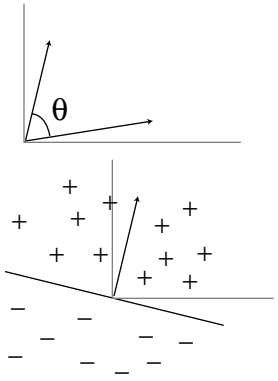
## dot product

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \bullet \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = x_1 x_2 + y_1 y_2$$

$$U \bullet V = \cos(\theta)\, \|U\|\, \|V\| \qquad \frac{U \bullet V}{\|U\|\, \|V\|} = \cos(\theta)$$

If $U \perp V$, then $U \bullet V = 0$

If $U \parallel V$, then $U \bullet V = \|U\|\, \|V\|$ or $-\|U\|\, \|V\|$

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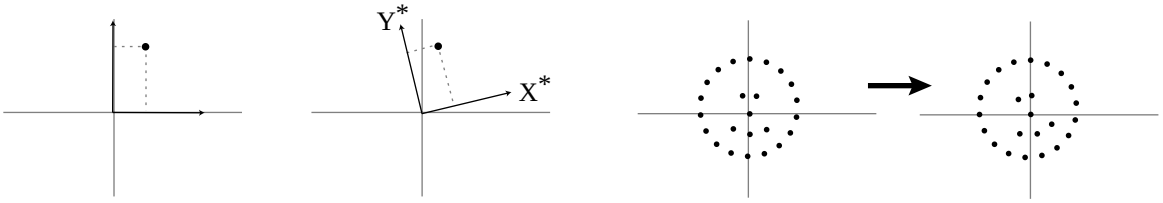
## rotation

$$X^* = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} \qquad Y^* = \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} @\theta = x_1 X^* + y_1 Y^* = \begin{bmatrix} x_1\cos(\theta) - y_1\sin(\theta) \\ x_1\sin(\theta) + y_1\cos(\theta) \end{bmatrix}$$

$$V@\theta = \begin{bmatrix} X^* & Y^* \end{bmatrix} V = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} V$$

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## interpolation

for k=0 to n (n+1 steps)

$$\begin{bmatrix} x_k \\ y_k \end{bmatrix} = \frac{k}{n} \begin{bmatrix} x_S \\ y_S \end{bmatrix} + \frac{n-k}{n} \begin{bmatrix} x_F \\ y_F \end{bmatrix}$$